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Stochastic pseudomonotone parabolic obstacle problem: well-posedness & Lewy-Stampacchia's inequalities

We study existence and uniqueness of solutions (u, ρ) to

$$\begin{cases} du - \operatorname{div} a(u, \nabla u)\, dt + \rho\, dt = f\, dt + G(u)\, dW(t) & \text{in } D \times \Omega_T, \\ u(t,0) = u_0 & \text{in } L^2(\Omega; L^2(D)) \\ u = 0 & \text{in } \partial D \times \Omega_T \\ u \geq \psi & \text{in } D \times \Omega_T \\ -\rho \geq 0 \text{ and } \langle \rho, u - \psi \rangle = 0 & \end{cases}$$

as well as Lewy-Stampacchia inequalities, where $D \subset \mathbb{R}^d$ is a bounded Lipschitz domain, $T > 0$ and $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space. The leading operator in our equation is a nonlinear, second order pseudomonotone operator of Leray-Lions type and the multiplicative noise term is given by a stochastic integral with respect to a Q-Wiener process. The obstacle ψ and the external force f are assumed to be random satisfying the ordered dual assumption

$$f - \partial_t \left(\psi - \int_0^{\cdot} G(\psi)\, dW \right) + \operatorname{div} a(\psi, \nabla \psi) = h^+ - h^-$$

for nonnegative functions $h^+, h^- \in L^{p'}(\Omega_T; W^{-1,p'}(D))$. Solutions of the penalised problem are provided by an approximation with a higher-order singular perturbation. Existence and uniqueness of solutions (u, ρ) as well as Lewy-Stampacchia inequalities are shown first for regular h^-, the general case is done by an approximation of h^-. This is a joint work with Y. Tahraoui, G. Vallet and A. Zimmermann.