Explicit Euler scheme: a new way of proof for the existence of solutions for singular parabolic SPDEs.

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For parabolic nonlinear SPDEs driven by space-time white noise it is well-known that the forward/explicit Euler scheme might not converge to the continuous solution, mainly due to the lack of moment bounds. However, in case of bounded nonlinear drift, the numerical solution is expected to remain controlled, thus opening the way for a proof by compactness argument. By extending this idea to derivative of bounded nonlinear drift, e.g. distribution, it is possible to prove that the forward/explicit Euler scheme will converge to an element, with Hölder regularity, which is a solution of a singular SPDE. With enough Hölder regularity of the solution, we can conclude the proof by uniqueness.
In this talk, I will present how to obtain enough compacity on the numerical solutions build by explicit Euler schemes.
Moreover, during all the presentation, I will present numerical simulations to illustrate a well-known paradigm « If engineers can build it, mathematicians can prove it ».